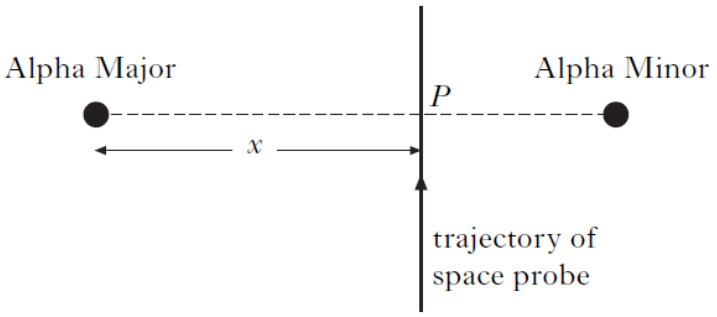
Gravitation

2007

A11. (*a*) The trajectory of a space probe passes between two planets Alpha Major and Alpha Minor which are L metres apart and with masses $4M$ kg and M kg respectively. The trajectory of the space probe intersects the line joining the centres of the planets at right angles at P, which is a distance x metres from Alpha Major.



The only forces acting on the space probe are the gravitational forces from the two planets. These forces obey the inverse square law. Assuming that these gravitational forces are equal and opposite, show that at P

$$3x^2 - 8xL + 4L^2 = 0.$$

Hence find the value of x, expressing your answer in terms of L. **5**

(*b*) The space probe is then manoeuvred into a circular orbit about Alpha Major at a height $2R$ metres above the surface of the planet, where R metres is the radius of Alpha Major.

Show that the period of the orbit is

$$T = \pi\sqrt{\frac{27R^3}{GM}},$$

where G is the universal constant of gravitation in its usual units. **5**

2012

A6. An International Space Station is held in a circular orbit by the Earth's gravitational field and travels at a height of 390 km above the surface of the Earth.

Given that the radius of the Earth is 6380 km, calculate:

(*a*) the speed of the satellite in its orbit; **4**

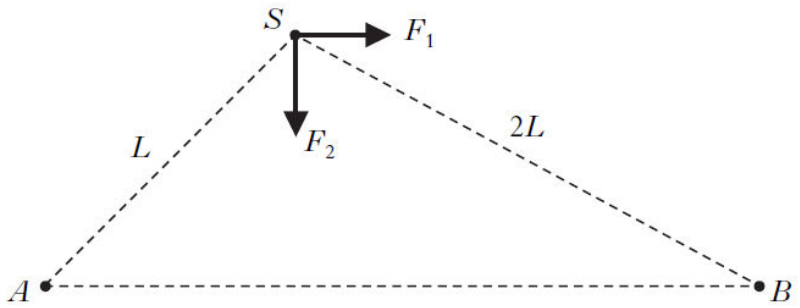
(*b*) the number of times the satellite orbits the Earth in one day. **3**

You should assume that Newton's Inverse Square Law of Gravitation applies.

2011

A7. A space probe S experiences a gravitational attraction from two asteroids, A and B. The mass of asteroid B is four times the mass of asteroid A.

At one instant, the probe is a distance L metres from asteroid A and distance $2L$ metres from asteroid B. The lines AS and BS are perpendicular. The force on the space probe has components F_1 and F_2 newtons, parallel and perpendicular to the line AB respectively as shown in the diagram.



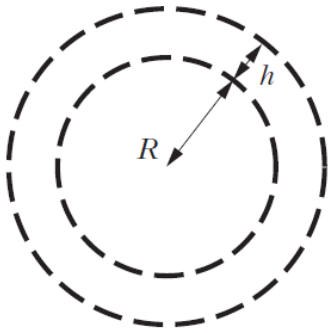
Assuming that the gravitational attraction on the probe due to each asteroid obeys the inverse square law, show that

$$F_2 = 3F_1.$$

6

2013

A11. A body of fixed mass m kilograms is projected vertically upwards from a point on the surface of a planet with an initial speed of u m s^{-1}.



Assuming that the gravitational force on the body is $\frac{GMm}{d^2}$ where d metres is the distance from the centre of the planet, show that the speed of the body when it has reached a height h metres above the surface is given by $v = \sqrt{u^2 - \frac{2GMh}{R(R+h)}}$, where M kilograms is the mass of the planet, R metres is the radius of the planet and G is the gravitational constant. **5**

Find an expression for the maximum height, H, reached by the body. **3**

Show that the escape speed necessary for the body to continue into space can be written in the form $u = k\sqrt{\frac{GM}{R}}$ and state the value of k. **2**

2015	**A5.** Two moons Bart and Casper complete circular orbits about a planet. Bart orbits with velocity v_B m s^{-1} and radius r metres. Casper orbits with velocity v_C m s^{-1} and radius $2r$ metres. (*a*) Show that $v_B = \sqrt{2}\,v_c$ and find the relationship between the angular velocities of the moons.	**4**
	(*b*) If Bart has a period of n days, find the period of Casper's orbit. You should assume that Newton's Inverse Square Law of Gravitation applies.	**2**
2016 SP	**13.** The distance of the Earth from the Sun is $1{\cdot}50 \times 10^{11}$ metres. The distance of Venus from the Sun is $1{\cdot}08 \times 10^{11}$ metres. Calculate the period of rotation of Venus around the Sun, giving your answer in Earth years. State **one** assumption you have made when calculating your answer.	**6**
2017	**13.** A satellite orbits the Earth at a height of h metres above its surface. (a) If the radius of the Earth is R metres and the acceleration due to gravity experienced at the surface of the Earth is 9 times that experienced at the satellite, find an expression for h in terms of R.	**4**
	(b) If a second satellite is orbiting Earth at a height $3R$ metres above the surface, show that the angular velocity of the second satellite can be expressed as $\frac{1}{8}\sqrt{\frac{g}{R}}$.	**3**